University of Arkansas at little Rock
Department of Systems Engineering

SYEN 3314 Probability and Random Signals - Summer 2009

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Midterm 1 - Wednesday, June 10, 2009

- This is a closed book exam.
- Calculators are not allowed.
- There are 8 problems on the exam plus one extra credit (or bonus) problem.
- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best.
- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly but concisely indicate your reasoning and show all relevant work. Your grade on each problem will be based on our assessment of your level of understanding as reflected by what you have written in the space provided.
- Please be neat and box your final answer, we cannot grade what we cannot decipher.

Name

Problem 1

A die is tossed. Find the probabilities of the events

1. $A = \{\text{odd number shows up}\}$
2. $B = \{\text{number larger than 3 shows up}\}$
3. $A \cap B$
4. $A \cup B$

Problem 2

Computer programs are classified by the length of the source code and by the execution time. Programs with more than 150 lines in the source code are big (B). Programs with $\leq$ 150 lines are little (L). Fast programs (F) run in less than 0.1 seconds. Slow programs (W) require at least 0.1 seconds. Monitor a program executed by a computer. Observe the length of the source code and the run time. The probability model for this experiment contains the following information: $P[LF] = 0.5$, $P[BF] = 0.2$, $P[BW] = 0.2$. What is the sample space of the experiment? Calculate the following probabilities:

1. $P[W]$
2. $P[B]$
3. $P[W \cup B]$

Problem 3

A short-circuit tester has a red light to indicate that there is a short circuit and a green light to indicate that there is no short-circuit. Consider an experiment consisting of a sequence of **three tests**. In each test the observation is the color of the light that is on at the end of a test. An outcome of the experiment is a sequence of red (r) and green (g) lights. Each outcome (a sequence of three lights, each either red or green) is **equally likely**. We denote each outcome by a three-letter word such as *rgr*, the outcome that the first and third lights were red but the second light was green. We denote the event that light n was red or green by R_n or G_n.

1. What is the sample space of the experiment?
2. Write the event R_2 that the second light is red
3. Write the event G_2 that the second light is green
4. Compute $P[R_2]$
5. Compute $P[G_2]$
6. Compute $P[R_2 \cap G_2]$

Are the events R_2 and G_2 independent? Are the events R_1 and R_2 independent?

Problem 4

You have a six-sided die that you roll once. Let R_i denote the event that the roll is i. Let G_j denote the event that the roll is greater than j. Let E denote the event that the roll of the die is even-numbered.

1. What is $P[R_3|G_1]$, the conditional probability that 3 is rolled given that the roll is greater than 1?

2. What is the conditional probability that 6 is rolled given that the roll is greater than 3?

3. What is $P[G_3|E]$, the conditional probability that the roll is greater than 3 given that the roll is even?

4. Given that the roll is greater than 3, what is the conditional probability that the roll is even?

Problem 5

Consider a binary code with 4 bits (o or 1) in each codeword. AN example of a code word is 0110.

1. How many different codewords are there?
2. How many codewords have exactly two zeros?
3. How many codewords begin with a zero?
4. In a constant-ratio binary code, each codeword has N bits. In every word, M of the N bits are 1 and the other $N - M$ bits are 0. How many different codewords are in the code with $N = 8$ and $M = 3$?

Problem 6

The random variable N has PMF

$$P_N(n) = \begin{cases} c/n, & n = 1, 2, 3; \\ 0, & \text{Otherwise.} \end{cases} \tag{1}$$

Find

1. the value of the constant c?
2. $P[N \geq 2]$
3. $P[N = 1]$
4. $P[N > 3]$

Problem 7

Suppose we test 10 circuits and each circuit is rejected with probability $p = 1/4$ independent of the results of other tests. What is the probability that there exactly 3 circuits rejected?

Problem 8

The number of hits at a Web site in any time interval is a Poisson random variable. A particular site has on average $\lambda = 2$ hits per second.

1. What is the probability that there are no hits in an interval of 0.25 seconds?
2. What is the probability that there are no more than two hits in an interval of one second?

Recall that X is a Poisson(α) random variable if the PMF of X has the form

$$P_X(x) = \begin{cases} \frac{\alpha^x e^{-\alpha}}{x!}, & x = 0, 1, 2, \cdots; \\ 0, & \text{Otherwise.} \end{cases} \tag{2}$$

where the parameter $\alpha = \lambda T$: λ is the average rate per second and T a time interval.

Extra credit problem worth 10 points

In an experiment with equiprobable outcomes, the event space is $S = \{1, 2, 3, 4\}$ and $P[s] = 1/4$ for all $s \in S$. Find three events in S that are pairwise independent but are not independent.